

Day 25

Extended Kalman Filter

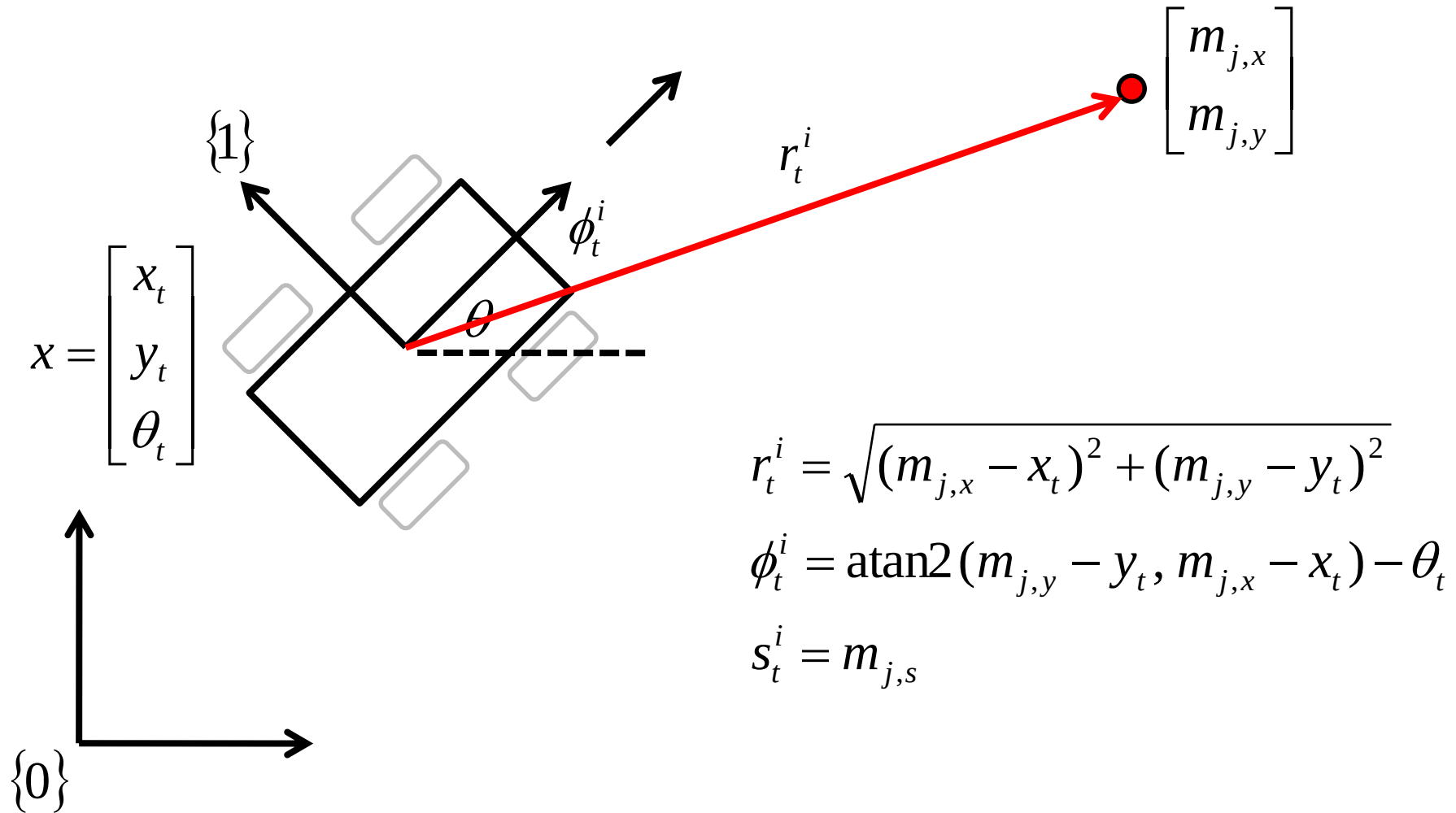
with slides adapted from <http://www.probabilistic-robotics.com>

Kalman Filter Summary

- **Highly efficient:** Polynomial in measurement dimensionality k and state dimensionality n :
$$O(k^{2.376} + n^2)$$
- **Optimal for linear Gaussian systems!**
- Most robotics systems are **nonlinear!**

Landmark Measurements

- distance, bearing, and correspondence



Nonlinear Dynamic Systems

- Most realistic robotic problems involve nonlinear functions

$$x_t = g(u_t, x_{t-1})$$

$$z_t = h(x_t)$$

Nonlinear Dynamic Systems

- localization with landmarks

$$x_t = \underbrace{\begin{pmatrix} x - \frac{v_t}{\omega_t} \sin \theta + \frac{v_t}{\omega_t} \sin(\theta + \omega_t \Delta t) \\ y + \frac{v_t}{\omega_t} \cos \theta - \frac{v_t}{\omega_t} \cos(\theta + \omega_t \Delta t) \\ \theta + \omega_t \Delta t \end{pmatrix}}_{g(u_t, x_{t-1})}$$

$$\underbrace{\begin{pmatrix} r_t^i \\ \phi_t^i \\ s_t^i \end{pmatrix}}_{z_t^i} = \underbrace{\begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ \text{atan2}(m_{j,y} - y_t, m_{j,x} - x_t) - \theta \\ m_{j,s} \end{pmatrix}}_{h(x_t, j, m)}$$

Linearity Assumption Revisited

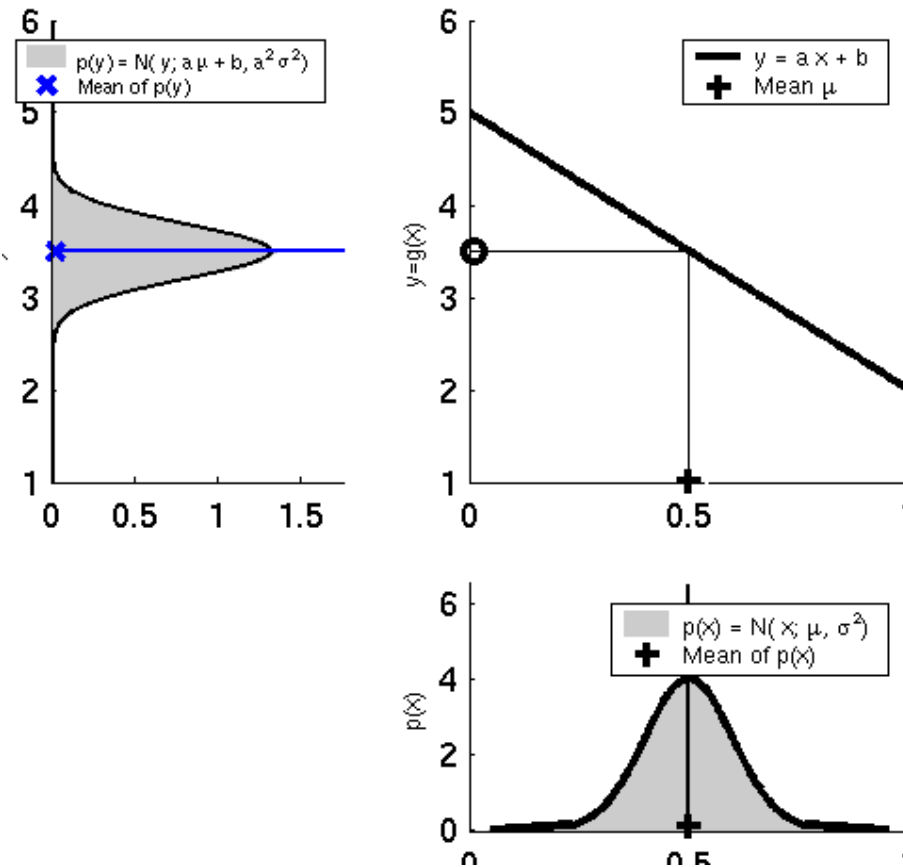


Figure 3.3 (a), p 55

Non-linear Function

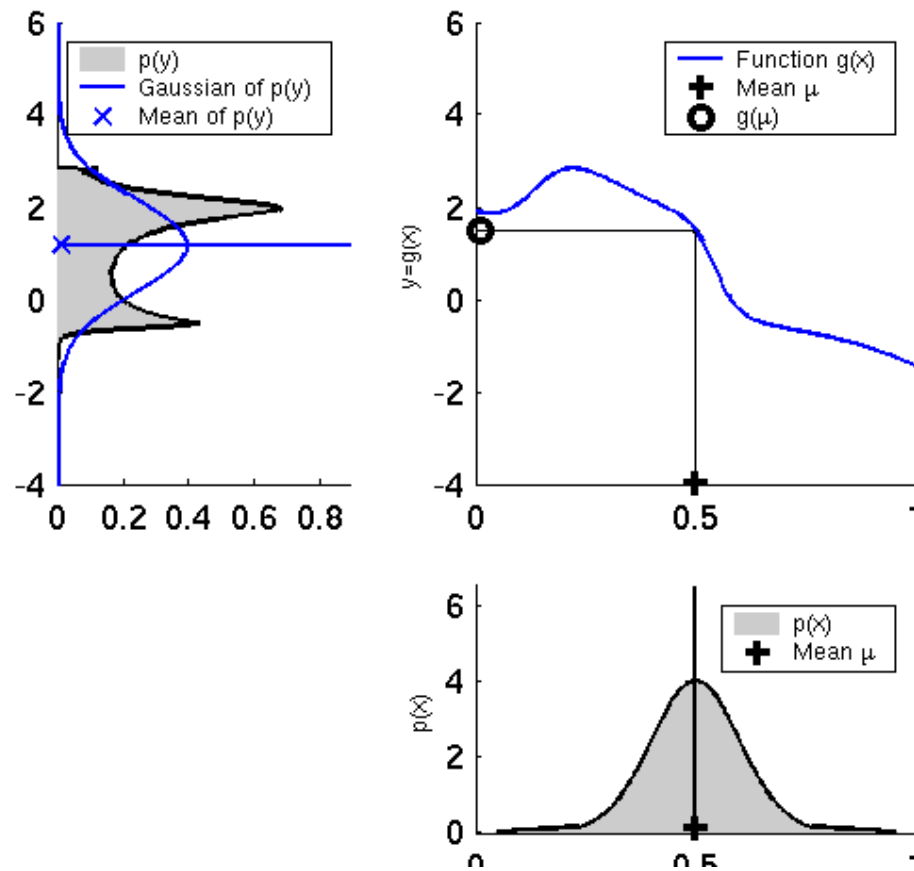


Figure 3.3 (b), p 55

EKF Linearization (1)

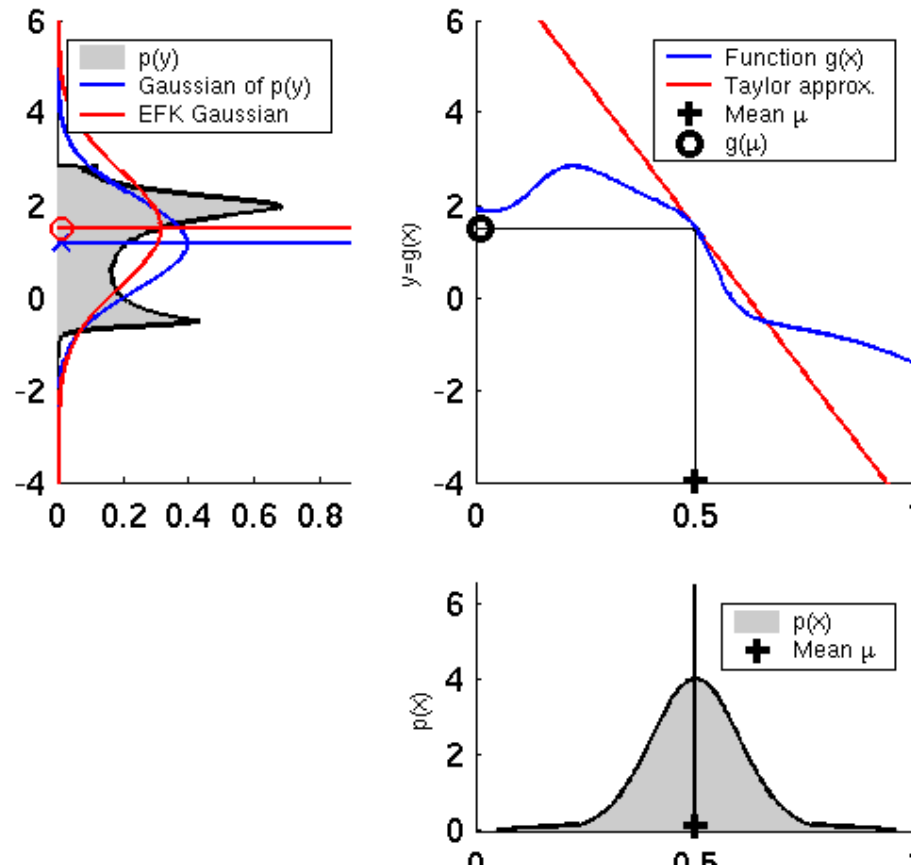


Figure 3.4, p 57

EKF Linearization (2)

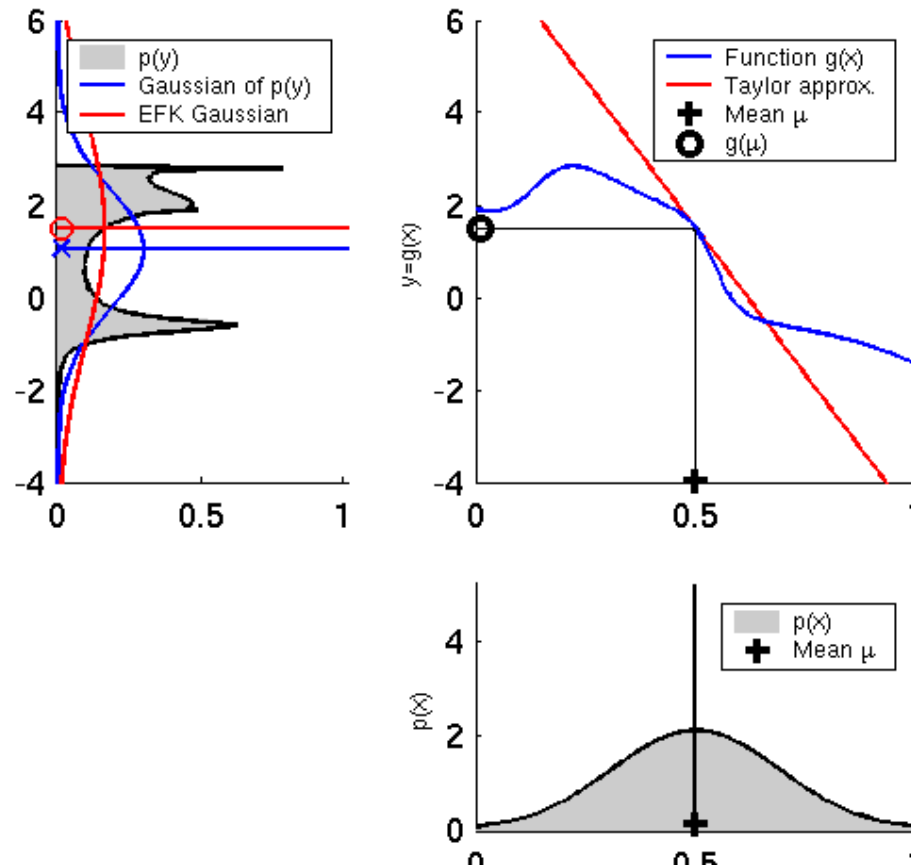


Figure 3.5 (a), p 62

EKF Linearization (3)

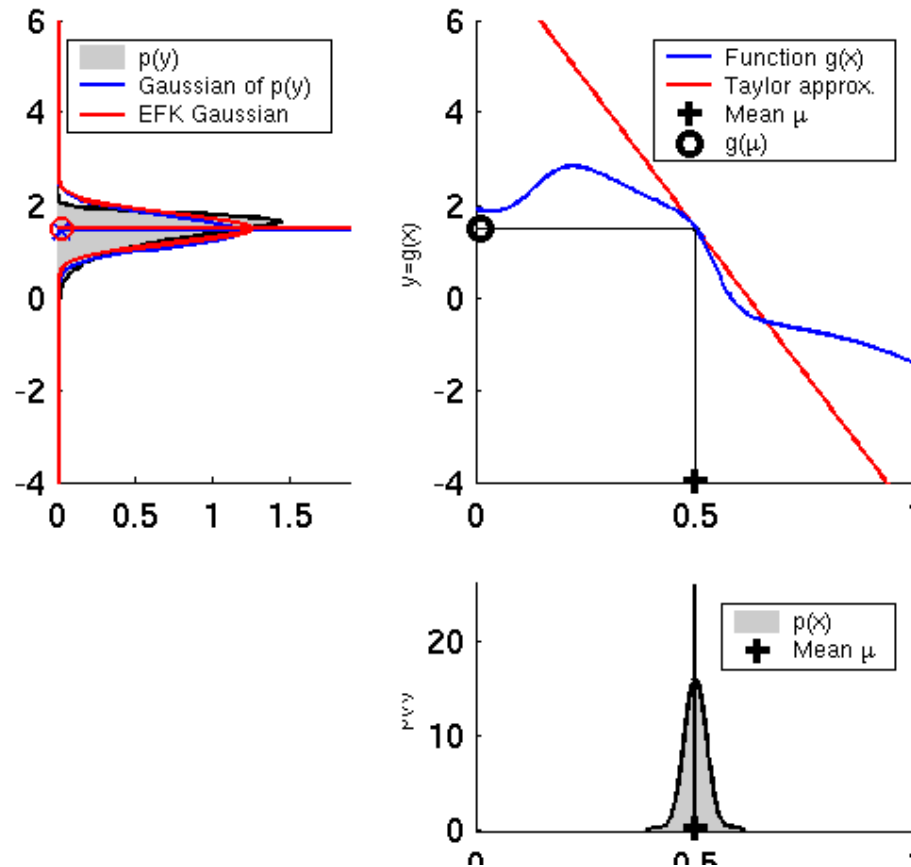


Figure 3.5 (b), p 62

Taylor Series

- recall for $f(x)$ infinitely differentiable around in a neighborhood a

$$\begin{aligned} f(x) &= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots \\ &\approx f(a) + f'(a)(x-a) \end{aligned}$$

- in the multidimensional case, we need the matrix of first partial derivatives (the Jacobian matrix)

EKF Linearization: First Order Taylor Series Expansion

- Prediction:

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} (x_{t-1} - \mu_{t-1})$$

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t (x_{t-1} - \mu_{t-1})$$

- Correction:

$$h(x_t) \approx h(\bar{\mu}_t) + \frac{\partial h(\bar{\mu}_t)}{\partial x_t} (x_t - \bar{\mu}_t)$$

$$h(x_t) \approx h(\bar{\mu}_t) + H_t (x_t - \bar{\mu}_t)$$

EKF Algorithm

1. **Extended_Kalman_filter**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):

2. Prediction:

3.	$\bar{\mu}_t = g(u_t, \mu_{t-1})$	$\longleftarrow$	$\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
4.	$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$	$\longleftarrow$	$\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

5. Correction:

6.	$K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$	$\longleftarrow$	$K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7.	$\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$	$\longleftarrow$	$\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
8.	$\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$	$\longleftarrow$	$\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. **Return** μ_t, Σ_t

$$H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t}$$

$$G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}}$$

Localization

“Using sensory information to locate the robot in its environment is the most fundamental problem to providing a mobile robot with autonomous capabilities.” [Cox '91]

- **Given**
 - Map of the environment.
 - Sequence of sensor measurements.
- **Wanted**
 - Estimate of the robot's position.
- **Problem classes**
 - Position tracking
 - Global localization
 - Kidnapped robot problem (recovery)

Landmark-based Localization



1. EKF_localization ($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, m$):

Prediction:

$$3. \quad G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} = \begin{pmatrix} \frac{\partial x'}{\partial \mu_{t-1,x}} & \frac{\partial x'}{\partial \mu_{t-1,y}} & \frac{\partial x'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial y'}{\partial \mu_{t-1,x}} & \frac{\partial y'}{\partial \mu_{t-1,y}} & \frac{\partial y'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial \theta'}{\partial \mu_{t-1,x}} & \frac{\partial \theta'}{\partial \mu_{t-1,y}} & \frac{\partial \theta'}{\partial \mu_{t-1,\theta}} \end{pmatrix} \quad \text{Jacobian of } g \text{ w.r.t location}$$

$$5. \quad V_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial u_t} = \begin{pmatrix} \frac{\partial x'}{\partial v_t} & \frac{\partial x'}{\partial \omega_t} \\ \frac{\partial y'}{\partial v_t} & \frac{\partial y'}{\partial \omega_t} \\ \frac{\partial \theta'}{\partial v_t} & \frac{\partial \theta'}{\partial \omega_t} \end{pmatrix} \quad \text{Jacobian of } g \text{ w.r.t control}$$

$$6. \quad M_t = \begin{pmatrix} (\alpha_1 |v_t| + \alpha_2 |\omega_t|)^2 & 0 \\ 0 & (\alpha_3 |v_t| + \alpha_4 |\omega_t|)^2 \end{pmatrix} \quad \text{Motion noise}$$

$$7. \quad \bar{\mu}_t = g(u_t, \mu_{t-1}) \quad \text{Predicted mean}$$

$$8. \quad \bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + V_t M_t V_t^T \quad \text{Predicted covariance}$$

1. EKF_localization ($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, m$):

Correction:

3. $\hat{z}_t = \begin{pmatrix} \sqrt{(m_x - \bar{\mu}_{t,x})^2 + (m_y - \bar{\mu}_{t,y})^2} \\ \text{atan2}(m_y - \bar{\mu}_{t,y}, m_x - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix}$ Predicted measurement mean

5. $H_t = \frac{\partial h(\bar{\mu}_t, m)}{\partial \mathbf{x}_t} = \begin{pmatrix} \frac{\partial r_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial \varphi_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial \varphi_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial \varphi_t}{\partial \bar{\mu}_{t,\theta}} \end{pmatrix}$ Jacobian of h w.r.t location

6. $Q_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_r^2 \end{pmatrix}$

7. $S_t = H_t \bar{\Sigma}_t H_t^T + Q_t$ Pred. measurement covariance

8. $K_t = \bar{\Sigma}_t H_t^T S_t^{-1}$ Kalman gain

9. $\mu_t = \bar{\mu}_t + K_t (z_t - \hat{z}_t)$ Updated mean

10. $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$ Updated covariance